Quantum Collision-Resistance of Non-Uniformly Distributed Functions

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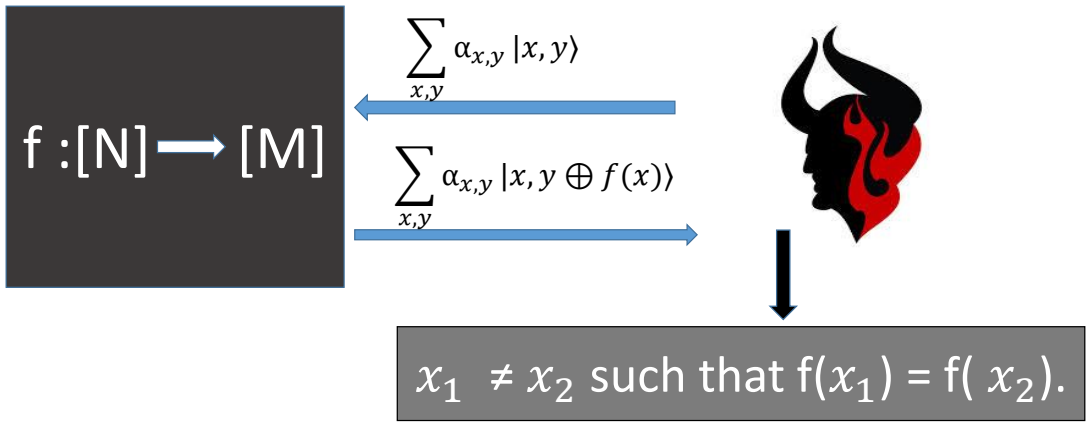
The problem: (Quantum Collision)



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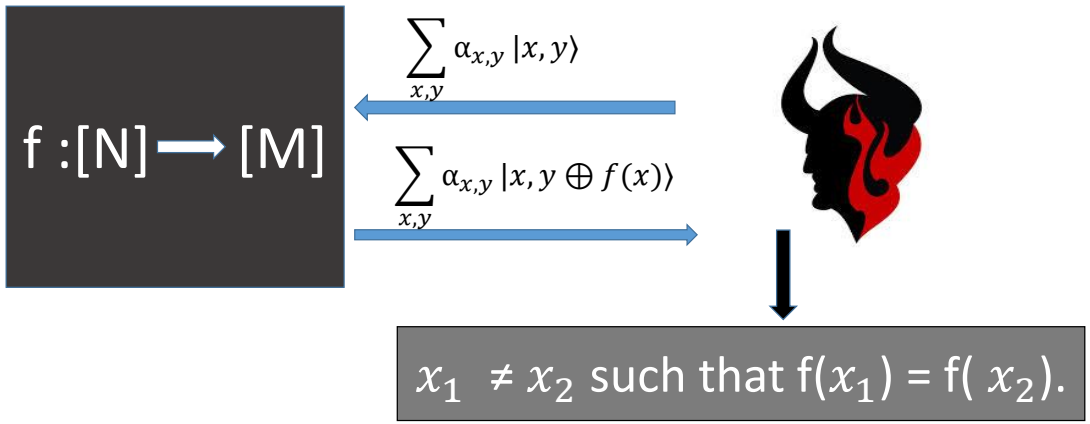

Question: How many quantum queries are needed to output a collision? (quantum query complexity point of view)
or
What is the maximum success probability given the specific number of queries? (quantum query solvability (Zhandry))

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Recall: By birthday attack, the probability of success is roughly $1/2$ when $q = \Theta(M^{1/2})$ for a classical adversary.

Previous result:

Theorem

Let $f : [N] \to [M]$ be a random function. Then any quantum algorithm making q number of queries to f outputs a collision for f with probability at most $\frac{C(q+2)^3}{M}$ where C is a universal constant.[1]

$\Rightarrow \Omega(M^{1/3})$ queries are needed to output a collision.

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Question: What if outputs of function f are chosen according to a non-uniform distribution.

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- In some cryptographic constructions, the combination of a truly random function and encryption function has to be collision-resistant:
  - Fujisaki-Okamoto transform:

$$Enc^{hy}_{pk}(m;\delta) = \left(Enc^{asy}_{pk}\left(\delta; H\left(\delta \| Enc^{sy}_{G(\delta)}(m)\right)\right),\ Enc^{sy}_{G(\delta)}(m)\right).$$

    - $Enc^{asy}_{pk} \circ H$ has to be collision-resistant.

Our contribution:

$$H_\infty(D) = -\log \max_{m \in [M]} \Pr[D(m)]$$

Theorem

Let $f : [N] \to [M]$ be a function whose outputs are chosen according to a distribution with min-entropy k. Then any quantum algorithm A making q queries to f returns a collision for f with probability at most $\frac{C'(q+2)^{9/5}}{2^{k/5}}$ where C' is a universal constant.

$\Rightarrow \Omega(2^{k/9})$ queries are needed to output a collision.

Preliminaries (for proof):

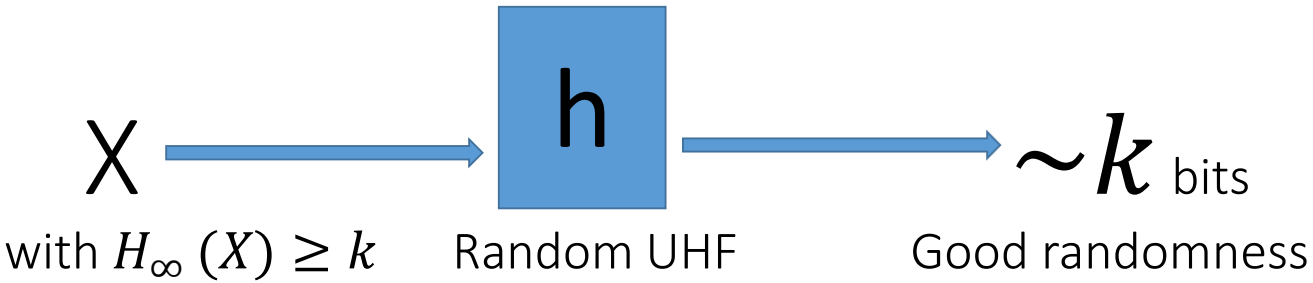
Definition (Universal Hash Function [2])

A family of functions $H = \{h : \{0,1\}^n \to \{0,1\}^m\}$ is called a universal family if for all distinct $x, y \in \{0,1\}^n$:

$$\Pr[h(x) = h(y) : h \xleftarrow{\$} H] \leq 1/2^m.$$

[2] [Larry Carter, Mark N. Wegman, J. Comput. Syst. Sci, 1979]

Preliminaries (Leftover Hash Lemma[3]):



[3] [Johan Håstad, Russell Impagliazzo, Leonid A. Levin, Michael Luby, 1993]

Proof sketch:

$$\Pr[\mathrm{Coll}(f; A_q^f) : f \leftarrow D^X]$$

$$\overset{(1)}{\leq} \Pr[\mathrm{Coll}(h \circ f; A_q^f) : f \leftarrow D^X]$$

$$\overset{(2)}{=} \Pr[\mathrm{Coll}(h \circ f; B_q^{h \circ f}) : f \leftarrow D^X] \text{ (preimages of } f \text{)}$$

$$\overset{(3)}{\approx} \Pr[\mathrm{Coll}(f^*; B_q^{f^*}) : f^* \xleftarrow{\$} \{\}] \text{ (LHL, } (D_1 \cong D_2 \iff D_1^X \cong D_2^X)^4)$$

$$\overset{(4)}{\approx} \text{ hard}$$

We prove that

$$\Pr[\mathrm{Coll}(f; A_q^f) : f \leftarrow D^X] \leq O(\frac{q^{9/5}}{2^{k/5}}).$$

[4] [Zhandry, How to Construct Quantum Random Functions, FOCS, 2012]

Question?

Thank you for listening!